Comparative analysis of the methods used in determination of half-life in a simple radioactive decay

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Precise and accurate determination of a radioactive half-life requires specialized experimental techniques as well as careful statistical analysis of the results. This report focuses on the latter. The analysis described below was applied to a set of simulated data that resembled those obtained from our recent measurement of the ^{46}V half-life. The current world-average value for the half-life is $t_{1/2}$ = 422.50(11) ms [1]).

In the actual measurement [2], ^{46}V ions were produced in the ^{47}Ti(p,2n)^{46}V reaction by a ^{47}Ti beam from the Texas A&M K-500 cyclotron. The ejectiles passed through the MARS recoil spectrometer and, in repeated cycles, a separated beam of ^{46}V was implanted into a 76-μm-thick aluminized mylar tape for ~1s, after which the collected radioactive sample was moved in 180 ms to the center of a 4π proportional gas counter for the detection of beta particles. Signals produced by the betas were processed and then counted by a pair of multi-channel scalers over 500 consecutive time intervals (channels) for a total of 10 s. The resulting spectra typically contained about 5000 counts, with average background on the order of 0.01 per channel. In a given run, the whole process was repeated for about 800 cycles, i.e., until a total of approximately four million pulses were counted by each scaler. The number of runs exceeded 30 and the experimental conditions were deliberately altered from run to run in order to test for any systematic dependence on those conditions.

In the study reported here, simulated rather than real data were used in order to avoid dealing with the effects of dead-time corrections and to be able to determine the accuracy of the fitted results for half-life and background since, for simulated data, we know precisely what those properties are. The amount of simulated data produced and their quality was comparable to the data collected in a single actual run. Specifically, we generated 1000 cycles of 500 channels each with a total of 4,500,000 counts using the input background value of 0.01 per channel per cycle and a simple exponential decay with a half-life of exactly 422.5 ms. No dead-time effects were included.

Results from three different methods of analysis (methods 1, 2, and 3) are compared below. Each method was applied both on a cycle-by-cycle basis and to the sum of all data. The former approach will be referred to as the “cycle fit”, while the latter will be referenced as the “sum fit”. The equations used for both approaches are summarized in Tables 2 and 3 of Koslowsky *et al.* [3]. In method 1 of our analysis we use a simple fitting procedure aimed at minimizing χ^2 given by

$$\chi^2 = \sum_{i=1}^{N} W_i (Y_i - D_i)^2 , \qquad (1)$$

where D_i and Y_i are the experimentally determined and the theoretically expected number of events in channel i, and W_i is the statistical weight of the i-th term, which in the present application equals the reciprocal value of Y_i. N is the total number of channels.

Method 2 is similar to method 1, except that W_i is assumed to equal 1 for all i. This method is often referred to as unweighted regression and it is the default method of analysis in the most common general-purpose spreadsheet programs such as Microsoft's Excel™ and SPSS' SigmaPlot™.

Method 3, on the other hand, uses a maximum likelihood approach. It involves maximization of the probability that the measured spectrum is a representation of the fitting function. Since the data follow Poisson statistics, this probability, which in this application is known as the maximum likelihood estimator, is given by

$$L = \prod_{i=1}^{N} \frac{e^{-Y_i} Y_i^{D_i}}{D_i!} . \qquad (2)$$

As shown in Table I, all three methods reproduced the correct value for the half-life using a sum fit approach. Compared to the other two methods, method 3 was slightly more accurate. Regarding the background, method 2 underestimates the input value by many of its reported standard deviations, while method 1 overestimates the expected result by nearly two of its. Method 3 produced the most accurate result, well within its reported error bar of the input value. The background error for method 3 is the largest among the three methods, but only by a small margin. For illustration purposes, the sum spectrum of the simulated data is shown in Fig. 1 along with curves representing the fitted results from method 3.

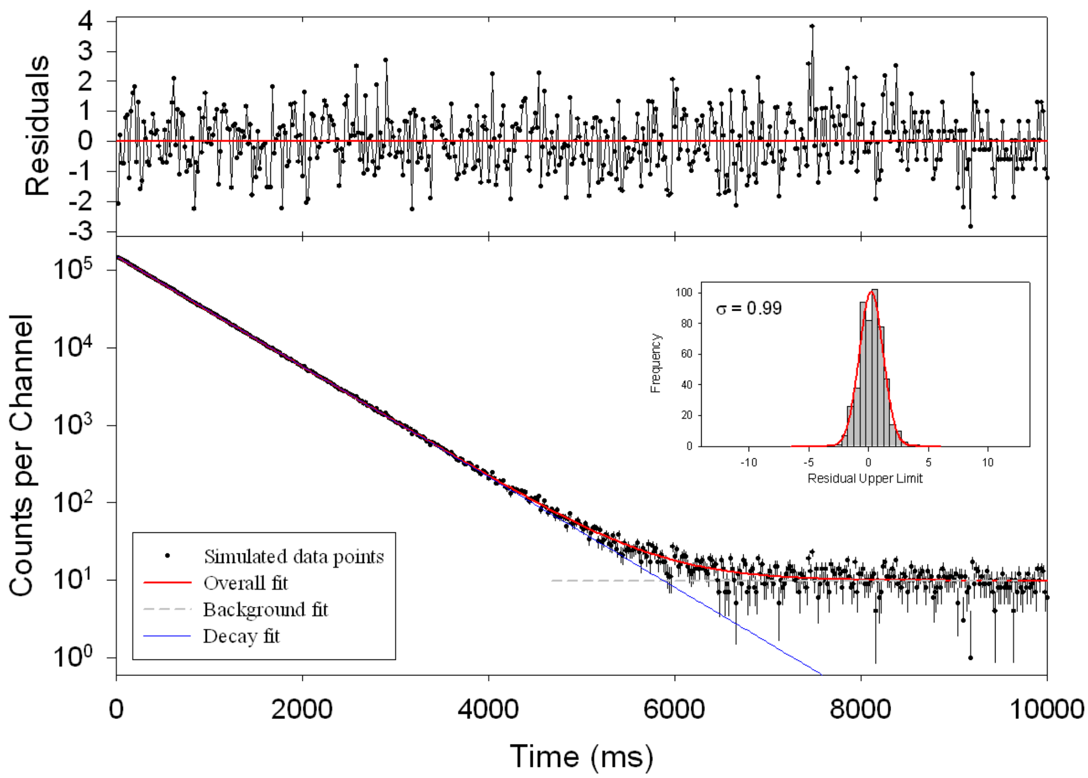

FIG. 1. Sum spectrum of the simulated data and the curves representing the fits with method 3 as described in the text.

Table I. Results of the sum-fit approach for the three methods described in the text applied to a set of simulated data with half-life 422.5s and a background of 10 counts per channel (in 1000 cycles).

Method	Half-life (ms)	Background per channel
1	422.41 ± 0.20	10.37 ± 0.20
2	422.68 ± 0.20	7.65 ± 0.15
3	422.42 ± 0.20	9.89 ± 0.22

Table II shows that in the cycle-fit approach method 1 produces completely erroneous results both for the background and the half-life. This can be traced to the fact that, in this method of fitting, the χ^2 [eq.(1)] can be reduced in two ways: either by a reduction in the absolute differences between the data points and their predicted values – the desired mechanism – or by a reduction in the statistical weights W_i (= 1 / Y_i), which tends to drive Y_i to higher, incorrect values. Of the other two methods, method 3 results in more accurate and more precise values of both the half-life and background compared to method 2.

Table II. Results of the cycle-fit approach for the three methods described in the text applied to a set of simulated data with half-life 422.5s and a background of 10 counts per channel (in 1000 cycles)

Method	Half-life (ms)	Background per channel
1	437.12 ± 0.25	124.7 ± 1.3
2	422.75 ± 0.37	7.8 ± 2.5
3	422.36 ± 0.19	9.95 ± 0.22

Since method 3 was found to be superior to the other two methods, it was also applied to a "global-fit" procedure [3], which is, in fact, a cycle-fit with a common value of the half-life that applies to all cycles. The resulting half-life value was 422.55(21) ms, which is accurate to about one part in 10,000, but the size of the error bar suggests that a result this good is just accidental. The error bar, apparently, was not reduced compared to that obtained in either the cycle fit or the sum fit. All three are effectively the same.

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[3] W. T. Koslowsky, E. Hagberg, J. C. Hardy, G. Savard, H. Schmeing, K. S. Sharma, and X. J. Sun, Nucl. Instrum. Methods Phys. Res. **A401**, 289 (1997).